How does Mass Affect the Efficiency of an Air Track Pulley System?

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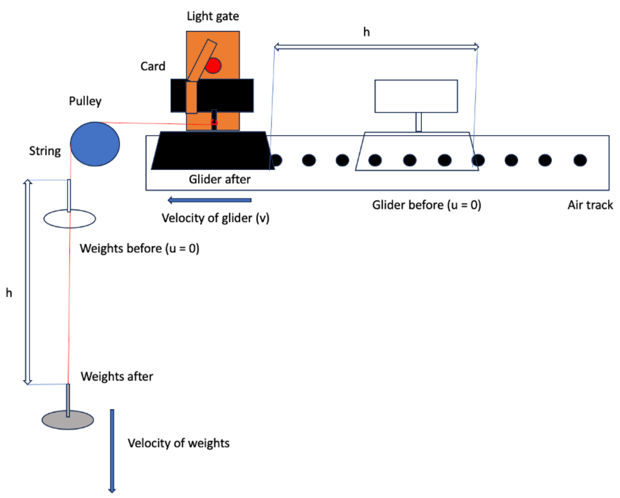
Abstract

Air tracks drastically minimize friction by injecting compressed air that escapes through the small holes on the track, providing a force strong enough to lift the gliders. As a result, it can provide a simulation of 'zero friction', and therefore they can effectively be used to test energy conversions. This led to the research question, how does the mass affect the efficiency of the conversion from the loss in gravitational potential energy (GPE) of the weights to kinetic energy (KE) gain of the glider in a pulley and air track system?

Keywords: Efficiency, GPE, KE

1. Diagram



2. Background Theory

According to the theory of conservation of energy:

$$GPE = KE$$

$$m_1gh = \frac{1}{2}m_2v^2$$

m_1 = the mass of the weights
m_2 = the mass of the glider

All of the gravitational potential energy of the weights should be converted to kinetic energy of the weights and the glider.

In reality:

$$GPE = KE\ (+\ wasted\ energy)$$

Energy can be lost through several ways, for example work done by friction between the glider and the air track, but also work done by friction between the string and pulley as the weights fall and drag/air resistance. Additionally, the air track still has a low coefficient of friction, which means there will still be some friction and so it will not match theoretical results.[1] More problems will be explained later.

2.1 Objective

The relationship between efficiency and mass is going to be investigated in this exploration to see how in the real world, large masses can affect the efficiency of large pulley systems, making it more inefficient. By doing this experiment, a trend or correlation can be discovered, and the efficiency will be quantifiable for different masses.

$$Efficiency = \frac{Useful\ energy\ output}{Total\ energy\ input} \times 100$$

$$Efficiency = \frac{KE\ gain}{GPE\ loss} \times 100$$

The velocity of the glider will be the same as the velocity of the weights, as the tension in the string makes them have the same acceleration, and therefore the same velocity, so only the velocity of the glider was measured. Then, the kinetic energy for the glider and the weights can individually be found.

[1] Lasserre, G. (2015)

2. Research Question: How does the mass affect the efficiency of the system?

Hypothesis: A larger mass will cause the system to have less efficiency due to greater frictional force between glider and air track and other factors.

Equipment List

- Air track
- Air blower
- Pulley
- Glider and card
- Weights (10g to 100g)
- String
- Light gate
- 2 clamp stands
- 1 T-clamp

Variables

Independent variable: Mass

The mass will range from 0g to 100g.

Dependent: Velocity

Velocity of the glider will be measured.

Control variable	**How it was controlled**	**Why is it being controlled**	**What is its value and uncertainty?**
Height the weights are dropped from	The bottom of the mass was held at the same initial height (the bottom edge of the table)	To ensure that the height remains the same, so GPE is the same.	0.61m± 0.001m
The glider used/its mass	Using the same glider	Changing the glider could change the air resistance, friction, mass – which would change the calculation for KE.	194.21g±0.01 g
The air track/its setting	The same air track was used, and its setting was kept the same	A different air track or a different setting could both change the results (increase/decrease friction)	

The pulley	The same pulley was used	A different pulley could cause a different amount of friction	

Safety and environmental concerns

Issues	**Issue with experiment**	**Preparation taken**
Safety	Moving the air track can be dangerous/hit others. Electricity/wires/circuits.	Carefully moved the air track far away from others. Only turned the air track on and plugged the wires when doing experiments.
Environmental	Noise pollution	Experiment was done far away from others

Diagram/setup

3. Methodology

<u>3.1 Setting up the equipment</u>

1. Weigh the glider on the balance
2. Set up the air track – adjust it so that it is not inclined
3. Set up the light gate a specific distance from the end of the air track and clamp it using the clamp stand
4. Put the pulley at the end of the air track and clamp it using a clamp stand. Use a T-clamp to clamp the clamp stand for more stability
5. Tie, using a string, the glider to the weights and place the string on the pulley. Make sure that the height of the pulley is around the same height as tied on the glider
6. Measure the height that the weight will drop from (0.61m)
7. Press go on the light gate

<u>3.2 Collecting data</u>

1. Turn the air blower on (keep the level of air blown constant)
1. Let go of the glider
2. Record the velocity
3. If needed, adjust the card on the glider to make it parallel to the air track
4. Bring back the glider to the same position on the air track
5. Repeat 3 times for each mass, masses from 10g to 100g to ensure that results are reliable and ignore anomalies

4. Data

Raw Data

<u>**Table 1 - Velocity for different masses**</u>

	Velocity (m/s) 0.001		
Mass (g) 0.01	1	2	3
10.00	0.709	0.704	0.709
20.00	0.990	0.990	1.020
30.00	1.190	1.190	1.219
40.00	1.330	1.350	1.369
50.00	1.470	1.470	1.492
60.00	1.538	1.754	1.538
70.00	1.666	1.724	1.754
80.00	1.754	1.786	1.754
90.00	1.923	1.886	1.851
100.00	1.960	1.960	1.978

I omitted trial 2 of mass 60g (highlighted in red) in the average and therefore the graph, as it is an anomaly.

Processed Data

Table 2 – Show average velocity, KE, GPE and efficiency for different masses

Mass (g)	Average velocity (m/s)	Kinetic Energy (J)	GPE (J)	Efficiency (%)
10.00	0.707	0.0511	0.0598	85.4
20.00	1.000	0.107	0.120	89.5
30.00	1.200	0.161	0.180	89.9
40.00	1.351	0.213	0.239	89.3
50.00	1.477	0.267	0.299	89.1
60.00	1.538	0.301	0.359	83.7
70.00	1.715	0.388	0.419	92.7
80.00	1.764	0.427	0.479	89.2
90.00	1.887	0.506	0.539	93.9
100.00	1.966	0.569	0.598	95.0

The result for 60.00g is an anomaly so it is not included in calculating the line of best fit.

Calculations carried out

For example, in column 1 (mass = 10.00g):

$$KE = \frac{1}{2}m_2v^2 = \frac{1}{2}\times\left(\frac{194.21+10}{1000}\right)\times 0.707^2$$

(194.21g is the mass of the glider)

$$KE \approx 0.0511 \text{ J}$$

$$GPE = m_1gh = \frac{10.00}{1000}\times 9.81\times 0.610$$

(Height that the weights dropped was 0.610m)

$$GPE \approx 0.0598 \text{ J}$$

$$Efficiency = \frac{KE\ gain}{GPE\ loss}\times 100 = \frac{0.0511}{0.0598}\times 100$$

$$Efficiency \approx 85.4\ \%$$

Table 3 – Show uncertainty of velocity and KE

Mass (g)	10.00	20.00	30.00	40.00	50.00	60.00	70.00	80.00	90.00	100.00
STDEV of velocity	0.003	0.017	0.017	0.018	0.013	0.125	0.045	0.018	0.036	0.010
Fractional uncertainty of velocity	0.004	0.017	0.014	0.013	0.009	0.078	0.026	0.010	0.019	0.005
Fractional uncertainty of v^2	0.008	0.035	0.028	0.027	0.017	0.155	0.052	0.020	0.038	0.011
Fractional uncertainty of KE of glider	0.008	0.035	0.028	0.027	0.017	0.155	0.052	0.020	0.038	0.011
Fractional uncertainty of KE of weights	0.009	0.035	0.028	0.027	0.017	0.162	0.052	0.020	0.038	0.011

Fractional uncertainty of KE

For example in column 1:

$$\frac{\Delta KE}{KE} = \frac{\Delta m}{m} + \frac{\Delta v}{v}\times 2$$

For glider

$$\frac{\Delta KE_{glider}}{KE_{glider}} = \frac{0.01}{194.21} + \frac{0.003}{0.707}\times 2$$

$$\frac{\Delta KE_{glider}}{KE_{glider}} = 0.008$$

For weights

$$\frac{\Delta KE_{weights}}{KE_{weights}} = \frac{0.01}{10.00} + \frac{0.003}{0.707}\times 2$$

$$\frac{\Delta KE_{weights}}{KE_{weights}} = 0.009$$

Uncertainties

Precision of measuring instruments

The light gate had a precision of 3 decimal places.

Uncertainty of velocity = ± 0.001m/s

The balance had a precision of 2 decimal places.

Uncertainty of mass of glider = ± 0.01g

Uncertainty of mass of weights = ± 0.01g

The metre rule had a precision of the nearest millimetre.

Uncertainty of height = ± 0.001m.

Table 4 – Fractional uncertainty of GPE and efficiency and absolute uncertainty of efficiency

Fractional uncertainty of GPE	0.003	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.002
Fractional uncertainty of KE	0.0004	0.0037	0.0045	0.0057	0.0046	0	0.0203	0.0087	0.0193	0.0060
Fractional uncertainty of efficiency	0.003	0.01	0.01	0.01	0.01	0.002	0.02	0.01	0.02	0.01

Fractional uncertainty of GPE
For example in column 1:

$$\frac{\Delta GPE}{GPE} = \frac{\Delta m_1}{m_1} + \frac{\Delta h}{h}$$

m_1 = the mass of the weights
(uncertainty of g is ignored)

$$\frac{\Delta GPE}{GPE} = \frac{0.01}{10.00} + \frac{0.001}{0.610}$$

$$\frac{\Delta GPE}{GPE} = \frac{3}{1000}$$

Fractional uncertainty of efficiency
For example in column 1:

Fractional uncertainty of kinetic energies of both the glider and the weights

$$\frac{\Delta KE}{KE} = \frac{0.008 \times \frac{1}{2} \times \frac{194.21}{1000} \times 0.707^2 + 0.009 \times \frac{1}{2} \times \frac{10}{1000} \times 0.707^2}{0.0511}$$

$$\frac{\Delta KE}{KE} = 0.00042$$

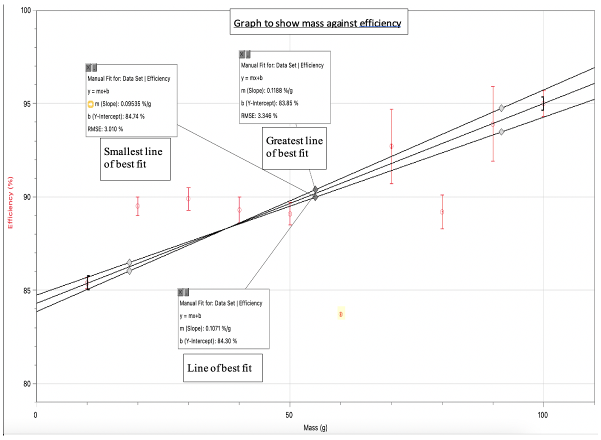
$$\frac{\Delta Efficiency}{Efficiency} = \frac{\Delta KE}{KE} + \frac{\Delta GPE}{GPE}$$

$$\frac{\Delta Efficiency}{Efficiency} = 0.00042 + 0.003$$

$$\frac{\Delta Efficiency}{Efficiency} = 0.00342$$

Analysis

Graph 1 – Mass against efficiency



Explanation of the Graph

This graph shows a positive correlation between mass and efficiency. This is shown in the equation of the line of best fit below:

$$y = mx + c$$

Efficiency = 0.107(mass) + 84.3%

This equation shows that as for every increase in 1 gram of mass, the efficiency increases by 0.107%. The results contradict my hypothesis, as it shows that the efficiency increased as mass increased.

The main explanation why my hypothesis could be wrong could be the fact that not all the energies in the system were considered:

$$GPE_{weights} = KE_{glider} + KE_{weights} + Rotational\ KE\ of\ pulley\ (+Energy\ lost)$$

As the mass increases, the velocities of the glider and the weights increase, causing an increase in their kinetic energies. For efficiency to increase instead of decrease, the increase in KE must be proportionally larger than the increase in gravitational potential energy. This is shown in the equation below:

$$Efficiency = \frac{KE\ of\ m_1\ and\ m_2 - loss - KE\ pulley}{GPE\ of\ m_1}$$

$$Efficiency = \frac{m_1 gh - loss - KE\ pulley}{m_1 gh}$$

Rotational KE of pulley = $\frac{1}{2} I\omega^2$, where I is the inertia and ω is the angular velocity.

$$Efficiency = 1 - \frac{loss}{m_1 gh} - \frac{I\omega^2}{2m_1 gh}$$

For efficiency to increase as mass increases, the $loss + KE\ pulley$ must be increasing at a slower rate than the increase in m_1. The loss here includes the work done by friction between the string and the pulley, the glider and the air track and the air resistance mainly of the weights, but also a very small amount from the glider.

Considering KE of pulley and there is no loss:

For cylinder: $I = \frac{1}{2} mr^2$, where m is the mass of the pulley and r is the radius

$$m_1 gh = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2} \times \frac{1}{2} mr^2 \left(\frac{v^2}{r^2}\right)$$

$$4m_1 gh = 2(m_1 + m_2)v^2 + mv^2$$

$$v^2 = \frac{4m_1 gh}{2(m_1 + m_2) + m}$$

$$Rotational\ KE\ of\ pulley = \frac{1}{4} mv^2$$

$$Rotational\ KE\ of\ pulley = \frac{mm_1 gh}{2(m_1 + m_2) + m}$$

$$Efficiency = 1 - \frac{KE\ of\ pulley}{m_1 gh}$$

$$= 1 - \frac{m}{2(m_1 + m_2) + m}$$

So we can see that as m_1 increases, efficiency increases.

As for the energy loss, we can consider the effect of air resistance in the equation below, where F_d is drag:

$$F_d = \frac{1}{2} \rho A v^2 C_d$$

For a disc:

$$C_d = 1.17$$

The radius of the mass hanger is approximately 2.5cm and the density of air as 1.225kg/m^3

When m_1 is 100g and $v = 1.966$, after substituting all the values, we get

$$F_d = \frac{1}{2} \times 1.225 \times \pi(0.025)^2 \times (1.966)^2 \times 1.17 = 5.44 \times 10^{-3}\ N$$

Assuming that acceleration is constant (which in reality it won't be), $F_d \propto v^2$, which $v^2 \propto s$, as u is 0:

$$v^2 = 2as$$

Then, we can find the work done by calculating the triangular area under the graph of force against displacement, which is the energy loss:

$$E_{loss} = \frac{1}{2} \times 5.44 \times 10^{-3} \times 0.61$$

$$E_{loss} = 1.66 \times 10^{-3}\ J$$

$$\%Loss = \frac{E_{loss}}{m_1 gh} \times 100 = \frac{1.66 \times 10^{-3}}{0.1 \times 9.81 \times 0.61} \times 100$$

$$\%Loss \approx 0.28\%$$

If we apply the same method with $m_1 = 10g$, we would obtain

$$\%Loss \approx 0.36\%$$

This shows that the percentage loss does increase with lower masses. However, work done by friction has not been considered in $\%Loss$. The coefficient of drag of the glider has also not been considered because it is extremely small (around 0.001).

Conclusion

In conclusion, even though the results contradicted my hypothesis, there are many reasons why.

At first, I assumed that as the velocities of the weights and glider increase, the work done by friction and air resistance would increase at a larger rate than the increase in mass. However, once I calculated it using the formulas, there is some evidence to show that efficiency should be increasing, just like the data showed in my experiment.

Evaluation

Problem	**Justification of relative importance**	**Type of error**	**Suggested improvement**	**Effect on final result**
The mass holder was 9.84g	After weighing the mass holder, I found that its mass was not 10g, but in fact smaller by 0.16g, which would impact all my results by a small amount.	This is a systematic error as the GPE loss calculated is less than it should be, meaning that the calculated efficiency is too high.	Use blue tack or some other material to add on to it/compensate for the weight.	The systematic error would be decreased, shifting the graph downwards
The pully was not always the same height level as glider	Although I tried to set them up at the same height in the beginning, the pulley was slipping and so the string wasn't always horizontal. This may cause a vertical vector instead of	This is a random error because the pulley changed heights for different readings.	Fix the pulley, make sure that it won't move by tightening the clamp. However, if this doesn't work, then attaching the air track pulley equipment may be a	The results will be more accurately as we are making sure that the glider only gains kinetic energy on a horizontal direction, not vertically, so the efficienc

	only being horizontal, which may be large depending on the angle.		better option.	y will be higher.
Friction between glider and air from air track/air track itself	The friction will most likely be a small amount	Random error – there will be different amounts of friction for each reading because the mass of the glider stays the same.	Sanding the edges and bottom of the glider to make it as smooth as possible	This could reduce the friction, which would increase the velocity and so there would be more efficiency.
Friction between string and pulley	The friction will likely be a small amount.	Random error (amount of friction changes) that will theoretically increase with each reading (as the mass/velocity increases)	Use the low friction pulley equipment for the air track.	This could reduce the friction, which would increase the velocity and so there would be more efficiency.

Card on glider tilted – so won't be 10cm.	This will affect the readings for the velocities by a small amount, as the card was	This is a random error because the card may be titled differently for	Make sure to adjust the card to be completely parallel to the air track	This would increase precision and the values for efficiency would

	not tilted that much. However, even if it was 1 cm off, the reading would be inaccurate by 10%.	each reading.	after each test.	decrease, as speed = distance/time, and the time would be longer if it recorded the whole card length, so the calculated speed of the glider decreases.
Weights not still/swinging	When the weights were moved up to its original position, they were sometimes still swinging when I started the experiment. However, this will affect the readings by quite a small amount as it is a low mass, and	Random error as weights weren't always swinging by the same amount/sometimes not swinging.	Make sure to keep the weights completely still before letting go of the glider.	This will make sure that the system has no motion of the weights to begin with so it will not affect the kinetic energy of the glider.

	the weights did not swing that much.			
Pulley direction was not aligned with air track perfectly	The pulley may be pulling the glider to a vector that is not parallel to the air track, which decreases efficiency. However, this will be by a small amount, as I tried to align them by eye.	Systematic error, as the pulley was always in the same position.	Use the attachable pulley equipment to put on the end of the air track.	This makes sure that the velocity vector of the glider is parallel to the air track, so efficiency values increase.
Inclination of air track	The air track was not completely horizontal, which may affect the velocity of the glider. However, this will have a small affect, as I	Systematic error as I did not adjust the settings of the air track.	Keep on adjusting the knobs until the air track is completely horizontal. This can be tested by comparing the air track to	The velocity of the glider will no longer be affected, so results will be more precise.

	attempted to adjust the air track already.		somethi ng horizont al.	

Precision and accuracy of measuring devices

The precision of the ruler is enough, as it can measure to the nearest millimetre, so it will only give a percentage error of 0.1%, meaning it is quite accurate. The light gate measures time to the nearest 3 decimal places (nearest millisecond), and the values of the velocities - which was repeated 3 times to be more reliable, but there could have been more repeats – were within an acceptable range of each other.

Further investigation

An interesting point about my results is that although the linear fit has the highest correlation for my data, this must not be the case, as the efficiency must level off at some point since it can’t keep on increasing; efficiency can never be above 100%, as in a closed system, the energy output cannot be greater than the energy input. So, a larger range of readings should have been done, for example increasing mass up to 300g. This may also reveal whether there is a relationship between mass and efficiency or not, as the correlation is not completely clear and the RMSE value is quite high. This may be a result of inaccuracies due to the setup of the experiment.

Also, the kinetic energy of the pulley could be calculated by using the formula $\frac{1}{2}I\omega^2$, so that the only inefficiency/loss in energy is from the work done by air resistance and friction. The experiment should also have more repeats, for example 5 repeats, to ensure more reliability.

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